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TOWARD A THEORY OF TRANSPORT IN HETEROGENEOUS MEDIA

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A unified equation is obtained for description of nonsteady state heat and mass transport in two-phase heterogeneous media in the low- and high-frequency approximations. Heating of a granular bed by a solid wall is considered as an example.

1. The basis of the traditional method of describing heat and mass transport processes in disperse and other heterogeneous materials is a system of equations for moderate temperatures or impurity concentrations in the individual phases, which consider interphase heat and mass exchange. Such equations are usually derived by the use of semiempirical relationships [1-3], although their general structure has been confirmed by results of a stricter analysis [4, 5]. We will write this system for heat transport in situations where convective and dispersive transport is significant for only one (continuous) phase:

$$\begin{aligned} \varepsilon d_1 c_1 \left(\frac{\partial}{\partial t} + u \nabla \right) T_1 &= \lambda_* \Delta T_1 - \beta (T_1 - T_2), \\ (1 - \varepsilon) d_2 c_2 \frac{\partial T_2}{\partial t} &= \beta (T_1 - T_2). \end{aligned} \quad (1)$$

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If the heterogeneous medium under consideration is macroscopically homogeneous, then ε , λ_* , and β are independent of coordinate; however, the usual assumption of time-independence of λ_* and β is a very strong assumption, which is approximately valid only in the limiting case of almost steady-state processes. Under nonsteady conditions this assumption is definitely not satisfied [6], and the question of the character of the approximation achieved by system (1) arises relative to its use for significantly nonsteady-state process.

Equations analogous to Eq. (1) in form and physical meaning have been used in many other situations involving simultaneous transport through coexisting contacting phases or components of a complex system of matter of some other physical nature (electrons, ions, etc.). We will note specially filtration transport of gas through a droplet liquid in fissured-porous and other materials with a dual porosity [7, 8], as well as filtration of a gas complicated by sorption effects [9, 10]. Such processes are also described by equations the structure of which is analogous to Eq. (1).

Even if we agree with the assumptions made in formulating Eq. (1), this system proves inconvenient for practical use. This fact has stimulated construction of models in the framework of which the heterogeneous medium is considered as some sort of continuum ("pseudo-liquid") with unique properties with formulation of a corresponding unique equation for the mean temperature (concentration) of such a continuum or one of the phases of the medium. An example of this approach is the introduction of a hyperbolic thermal conductivity (diffusion) equation in [11-13], which is justified to some degree by considerations following from the thermodynamics of irreversible processes [14-16].

A direct simplification of system (1) is also possible, within the limits of its physical accuracy, by using a representation of the quantity T_2 from the second equation as a result of action of a defined operator on T_1 with subsequent use of an operator expansion in powers of $(\partial/\partial t)^n$. Substitution of the approximate expression obtained for T_2 in the first equation leads to a unique, so-called equivalent, equation for T_1 , of the elliptic type. For system (1) this equation has the form

$$dc \frac{\partial T_1}{\partial t} + \varepsilon d_1 c_1 \mathbf{u} \nabla T_1 = \lambda_* \Delta T_1 + m \frac{\partial^2 T_1}{\partial t^2}, \quad (2)$$

$$dc = \varepsilon d_1 c_1 + (1 - \varepsilon) d_2 c_2, \quad m = (1 - \varepsilon)^2 (d_2 c_2)^2 / \beta.$$

An analogous equivalent equation for filtration processes was obtained in [8], while its generalization to situations where transport through both phases of the heterogeneous medium is significant was performed in [18].

Strict derivation of a unique equation can in principle be accomplished by ensemble averaging of local convective thermal conductivity (diffusion) equations in both phases and use of the fundamental concept of the self-consistent field theory, which makes possible consideration of collective effects present in the system, reflecting interaction of particles of the dispersed phase [5]. As a result, in the approximation under consideration, corresponding to processes close to steady state, one again obtains an equation which usually is of the elliptical type [6]. Although this method allows us to successfully describe the effect on the transport process of internal sources or drains, the kinetics of transport through the interphase boundary, and the effect of conductivity through the dispersed phase (see, for example, [19, 20]), it is quite laborious and cumbersome, and the results which it produces are usually poorly expressed.

Thus, the various approaches and model considerations lead to significantly different equations for description of heat and mass transport. Solutions of some of these equations have been evaluated and compared among themselves and with available experimental data in [21-23]. From the results of these evaluations there follows the general conclusion that the lack of clear physical concepts of the way the process occurs, as well as detailed experimental data, at present complicates choice of the most adequate model for engineering calculations. Under such conditions it becomes desirable to perform a direct analysis of transport processes at the level of individual elements of the dispersed phase on the basis of strict methods of the type developed in [4-6, 19, 20] without introduction of any poorly proved a priori assumptions. However, such methods should first be applied to special cases where it is possible to effectively use some simplifying assumptions, thus obtaining relatively simple and easily represented results.

2. In the general case the dispersion of the coefficients λ_* and β in Eq. (1) is caused by two basic relaxation processes: relaxation of the mean heat or mass flux to its steady-state value, corresponding to the given instantaneous value of the temperature or concentration gradient, and relaxation of the temperature (concentration) of the dispersed phase to the value of this quantity in the continuous phase, caused by the interphase heat (mass) transport mechanism. Both these processes are controlled by relaxation of the temperature or concentration fields both within the particles and in the spaces between them, while the inhomogeneity of the temperature or concentration distribution of the continuous phase along the surfaces of individual particles of the dispersed phase also plays a role. Significant simplification can be achieved if for some reasons the characteristic time of field relaxation in the continuous phase is much less than the corresponding time for particles, and it also becomes possible to neglect inhomogeneity of those fields over linear lengths of the order of the particle size. In this case it is possible to take the temperature (concentration) on the surface of each particle as independent of coordinate and coinciding with the mean for the continuous phase at the point of particle location, assuming the thermal diffusivity coefficient $\kappa_* = \lambda_*/d_1c_1$ (diffusion coefficient D_1) in Eq. (1) equal to its steady-state value, which is assumed known. Then in general there is no need to solve the complex boundary problem for heat or mass transport in the vicinity of a selected (test) particle [5, 6, 19, 20], and the problem reduces in fact to analysis of the dynamics of temperature or concentration change only within the particle. Analyses based on similar assumptions have been used previously several times (see, for example, [24, 25]).

The temperature relaxation times within particles and the intervals between particles are of the order of the quantities $\Delta t = a^2/\kappa$ and $\Delta t' = \ell^2/\kappa'$, respectively. Therefore, the above requirement on these times reduces to the inequality $\kappa' \gg (\ell/a)^2\kappa$. For granular particle layers having no intense anisotropy of form, the linear scale ℓ of the space between particles is at least $10^{1/2}$ -10 times less than the particle size a . Thus, the inequality takes on the form $\kappa' \gg (0.01 - 0.01)\kappa$ and for real materials proves valid for the majority of cases. The same situation occurs in mineral extraction thermophysics, since the characteristic size of fissures ℓ is much greater than the linear scale a of the monoliths or porous blocks separating them. However, for filtration in capillary-porous bodies with topologically similar phases usually $\ell \sim a$, i.e., the approximation under consideration is invalid.

The conditions for satisfaction of the inequality $\Delta t' \ll \Delta t$ are eased even more when we deal with diffusion of an impurity in a filtration flow, since the diffusion coefficient D' in the filtering liquid or gas exceeds by an order of magnitude or more the diffusion coefficient D in the porous particles being flowed over. An analogous inequality is almost always satisfied for filtration in fissured-porous media, when $\Delta t'a \sim \ell/v$, where v is the velocity of sound in the fluid and $\Delta t \sim a^2/\kappa$, where κ is the piezoconductivity coefficient in the porous blocks, while it is always true that $\ell \ll a$, $v \gg \kappa/a$.

Temperature or concentration inhomogeneity on the surface of an individual particle can be neglected approximately, if the inequality $a \ll L$ is satisfied, where L is the linear scale of the mean temperature or concentration fields. Since this same inequality is a necessary condition for applicability in principle of continuum methods for description of transport processes in heterogeneous media [in particular, the applicability of equations of the type of Eq. (1) or (2)], below we will consider it to be satisfied.

3. We will model the real particles of a granular layer by spheres of radius a , assuming that on the sphere surface the temperature $T_1 = T_0 \cos \omega t$ is satisfied. Within the sphere the temperature, satisfying the condition of periodicity necessary in the case under consideration, is defined by the expression [26]:

$$T = A(a/r) \cos(\omega t + \varphi), \quad \omega' = \sqrt{\omega/2\kappa}, \quad (3)$$

$$A = \left(\frac{\operatorname{ch} 2\omega'r - \cos 2\omega'r}{\operatorname{ch} 2\omega'a - \cos 2\omega'a} \right)^{1/2}, \quad \varphi = \arg \frac{\operatorname{sh}[(1+i)\omega'r]}{\operatorname{sh}[(1+i)\omega'a]}$$

[the term in Eq. (3) produced by the initial temperature distribution need not be considered in connection with the replacement of the initial condition by the periodicity condition].

We will consider Eq. (3) in the low-frequency approximation where the inequality $\delta = \omega'a \ll 1$ is satisfied. In this case

$$A = \frac{r}{a} \left[1 - \frac{1}{45} \omega'^4 (a^4 - r^4) + O(\delta^8) \right],$$

$$\varphi = -\frac{1}{3} \omega'^2 (a^2 - r^2) + O(\delta^6),$$

so that for the temperature we have

$$T = T_0 \left[1 - \frac{\omega^2 (a^4 - r^4)}{180\kappa^2} \right] \cos \left[\omega t - \frac{\omega (a^2 - r^2)}{6\kappa} \right] + O(\delta^6). \quad (4)$$

Hence it is simple to calculate the thermal flux to the particle

$$q = 4\pi a^2 \lambda \left. \frac{\partial T}{\partial r} \right|_{r=a} = \frac{4\pi a^3}{3} d_2 c_2 T_0 \left(-\omega \sin \omega t + \frac{\omega^2 a^2}{15\kappa} \cos \omega t \right) + O(\delta^6).$$

The expressions presented above for T and q for an ω -dependent value of T_0 can be considered as Fourier transforms of the temperature within the particle and the heat flux to the particle, dependent on time in an arbitrary manner. Applying a reverse Fourier transform to the latter expression, for the flux we obtain

$$q = \frac{4\pi a^3}{3} d_2 c_2 \left(1 - \frac{a^2}{15\kappa} \frac{\partial}{\partial t} \right) \frac{\partial}{\partial t} T_1 \quad (5)$$

(this expression, as all those below, is written to the accuracy of terms of the order of $\delta^4 \sim (\omega a^2 / \kappa)^2$ inclusive; the same notation is used for the original quantities and their Fourier transforms). Taking as the mean temperature of the dispersed phase at a given point the result of averaging the value of T , the Fourier transform of which is given by Eq. (4), over the volumes of a sphere with center at the given point, we obtain

$$4\pi d_2 c_2 \frac{\partial}{\partial t} \int_0^a T r^2 dr = \frac{4\pi a^3}{3} d_2 c_2 \frac{\partial T_2}{\partial t} = 4\pi a^2 \lambda \left. \frac{\partial T}{\partial r} \right|_{r=a} = q,$$

whence with consideration of Eq. (5) we obtain

$$T_2 = \left(1 - \frac{a^2}{15\kappa} \frac{\partial}{\partial t} \right) T_1. \quad (6)$$

The obvious thermal balance equation for the continuous phase can be written in the form

$$\varepsilon d_1 c_1 \left(\frac{\partial}{\partial t} + \mathbf{u} \nabla \right) T_1 = \lambda_* \Delta T_1 - nq, \quad n = \frac{3(1-\varepsilon)}{4\pi a^3},$$

whence with consideration of Eq. (5) it follows that

$$dc \frac{\partial T_1}{\partial t} + \varepsilon d_1 c_1 \mathbf{u} \nabla T_1 = \lambda_* \Delta T_1 + m \frac{\partial^2 T_1}{\partial t^2}, \quad (7)$$

$$m = (1-\varepsilon) d_2 c_2 (a^2 / 15\kappa),$$

while the heat capacity dc per unit volume of the heterogeneous dispersed medium is defined by Eq. (2). It will be convenient to rewrite Eq. (7) in the form

$$\frac{\partial T_1}{\partial t} + \mathbf{w} \nabla T_1 = \gamma^2 \Delta T_1 + \tau \frac{\partial^2 T_1}{\partial t^2}, \quad (8)$$

$$\mathbf{w} = \frac{\varepsilon d_1 c_1}{dc} \mathbf{u}, \quad \gamma^2 = \frac{\lambda_*}{dc}, \quad \tau = \frac{(1-\varepsilon) d_2 c_2}{dc} \frac{a^2}{15\kappa},$$

where \mathbf{w} , γ^2 , and τ play the role of the effective rates of convective heat transport, the thermal diffusivity coefficient, and the temperature relaxation time for low-frequency processes, respectively.

In form Eq. (7) coincides precisely with the equivalent Eq. (2), but it is important to find the relationship between the corresponding values of the coefficient m . For this purpose we write the interphase heat flux per unit volume in the form

$$nq = \beta_* (T_1 - T_2) = \beta_* \frac{a^2}{15\kappa} \frac{\partial}{\partial t} T_1 = (1 - \varepsilon) d_2 c_2 \left(1 - \frac{a^2}{15\kappa} \frac{\partial}{\partial t} \right) T_1$$

[where we consider Eqs. (5), (6)]. Hence it follows that β_* is the operator

$$\beta_* = \frac{15(1 - \varepsilon)}{a^2} \lambda \left(1 - \frac{a^2}{15\kappa} \frac{\partial}{\partial t} \right), \quad (9)$$

but with the heat-exchange coefficient by no means constant, as was assumed in formulating Eq. (1). In a coarser approximation, neglecting terms of order $\delta^4 \sim (\omega a^2/\kappa)^2$, the derivative in Eq. (9) can be dropped, thus identifying the coefficient β in Eq. (1) with $15(1 - \varepsilon)\lambda/a^2$. Then the parameter m introduced in equivalent Eq. (2) is exactly equal to m from Eq. (7), i.e., Eqs. (2) and (7) coincide completely. However, the theory leads just to Eq. (7), not at all to the phenomenological system (1). Therefore, the fact that use of the indicated system leads to good results in practical calculations in the case where the time scale of the process is sufficiently long is the result of a circumstance which is to a significant degree accidental. In fact this system was obtained with a much more coarse assumption than Eq. (7). However, it proves to be the case that, as was proved in [17], to the accuracy of terms of the order of δ^4 its solution coincides decisively with the solution of the equivalent equation for the corresponding problem of initial and boundary conditions. Thus, the equivalent equation in the form of Eq. (2), (7), or (8) proves to be physically more correct than the semiempirical system (1).

Equation (8) is of the elliptical type, and there is no difference in principle between the temporal and spatial independent variables appearing therein. As is well known, boundary conditions for correct formulation of various boundary problems on the basis of this equation must be specified over the entire boundary of the integration region. For the Dirichlet and Neiman problems this means that one must specify values of T_1 or its normal derivatives on all portions of the surface limited by the spatial region of definition of the equation, together with T_1 or its time derivative at the initial and final moments. The third and mixed boundary problems must be formulated similarly. After definition of T_1 the temperature T_2 is found by simple differentiation from Eq. (6).

According to [21-23], for sufficiently high Fourier numbers (sufficiently low frequency) the solutions of Eq. (8) correspond well to experiment as, incidentally, do the solutions of other proposed equations, including the conventional parabolic one. However, on the whole, the available experimental data is insufficient to indicate the boundaries of applicability of this theory in the range of small Fourier numbers where this theory (because of the use of the low-frequency approximation) can be considered only as very approximate. Nevertheless, we have sufficient experimental confirmation of the adequacy of the theory developed from analysis of the processes of heating and cooling of mineral masses and fissured plates with a fluid flux passing through them. Without going into details, we will note that for analysis of processes of this type Eq. (8) with $\gamma = 0$ was proposed previously by Smirnova [25], who compared its solution with a generalized complex of experimental data and found good agreement of the theory with these data.

4. We will now consider the high-frequency asymptote of Eq. (3), corresponding to the inequality $\delta = \omega' a \gg 1$. Such a situation is completely realistic in cases where penetration of heat or the impurity into the particles of the dispersed phase is difficult (the value of κ is small). We have

$$A \approx \exp[-\omega'(a-r)], \quad \varphi \approx -\omega'(a-r)$$

and further, in place of Eq. (4)

$$T = T_0(a/r) \exp[-\omega'(a-r)] \cos[\omega t - \omega'(a-r)], \quad (10)$$

while for the heat flux into the particle we obtain ($\omega a^2/\kappa \gg 1$)

$$q = 2\pi \sqrt{2\kappa a^2 d_2 c_2} T_0 \sqrt{\omega} (\cos \omega t - \sin \omega t).$$

As before, considering this as an expression for the Fourier harmonic of the flux q , dependent on time in an arbitrary manner, and integrating over all frequencies, we find (see the analogous transformations in deriving the expression for the Basse viscous force in [27, p. 131]) that

$$q = 4a^2 \sqrt{\pi \kappa d_2 c_2} \int_{-\infty}^t \frac{\partial T_1}{\partial t} \Big|_{t=t'} \frac{dt'}{\sqrt{t-t'}} \quad (11)$$

As a result, in place of Eq. (6) we obtain

$$\frac{\partial T_2}{\partial t} = \frac{3}{a} \sqrt{\frac{\kappa}{\pi}} \int_{-\infty}^t \frac{\partial T_1}{\partial t} \Big|_{t=t'} \frac{dt'}{\sqrt{t-t'}} \quad (12)$$

and in place of Eq. (7) or (8), the integrodifferential equation

$$\frac{\partial T_1}{\partial t} + u \nabla T_1 = \sigma^2 \Delta T_1 - \frac{1}{V \theta} \int_{-\infty}^t \frac{\partial T_1}{\partial t} \Big|_{t=t'} \frac{dt'}{\sqrt{t-t'}} \quad (13)$$

$$\sigma^2 = \frac{\lambda_*}{\varepsilon d_1 c_1}, \quad \frac{1}{V \theta} = \frac{3(1-\varepsilon)}{\varepsilon a} \sqrt{\frac{\kappa}{\pi}} \frac{d_2 c_2}{d_1 c_1},$$

where σ^2 and θ have the sense of effective thermal diffusivity and a new relaxation time, appearing in high-frequency transport processes, with respect to which the heterogeneous medium behaves like a material with memory; the possibility of such behavior was indicated in [28].

In real transport processes, accompanied by establishment of an equilibrium or steady state, the characteristic time scale usually increases monotonically, i.e., some continuous transition from the high-frequency asymptote described by Eqs. (12) and (13) to the low-frequency one of Eqs. (6) and (8) is accomplished. It can easily be seen that as the process develops the effect of thermal memory degenerates, while the intensity of both components of the thermal flux in the medium (the convective and the effective dispersion) decreases. In fact, $u > w$ and $\sigma^2 > \gamma^2$. We recall that for adequacy of hf approximation being used it is necessary that $a \ll L$; depending on which transport component predominates, for the linear temperature scale we have $L \sim u/\omega$ or $L \sim (\sigma^2/\omega)^{1/2}$.

All the results obtained are valid to an equal degree for description of convective-diffuse mass transport processes if we take $d_1 c_1 = d_2 c_2 = 1$ and understand by κ and λ_* the diffusion coefficient into particles and the effective impurity dispersion coefficient (also including molecular diffusion) in the continuous phase, respectively. An example of such a process would be moisture transport in the form of water vapor in nodular soil.

5. As an illustrative example we may consider heating of a layer of granular material with spaces filled by an immobile gas or liquid by a solid wall bordering the layer. We will assume the initial (at $t = 0$) temperatures of both phases of the layer equal to zero; for $t > 0$ the wall temperature is constant and equal to T^0 . We will consider the one-dimensional problem where T_1 and T_2 depend solely on the coordinate x normal to the wall with boundary conditions of the first sort at $x = 0$.

It is clear that the time scale of the process is of the order of magnitude of t , i.e., it changes with time from zero to infinity, the latter corresponding to establishment of a steady state for any finite region of space. The process is characterized by two relaxation times: $\theta \sim (a^2/\kappa)(d_1 c_1/d_2 c_2)^2$ and $\tau \sim 0.1 a^2/\kappa$ [see Eqs. (8) and (13)]. If there is a drop-let liquid within the layer then τ is approximately an order of magnitude less than θ ; if the layer is filled by gas then τ is several (four or more) orders of magnitude greater than θ . In the initial heating stage ($t \leq \tau$) one must use Eqs. (12) and (13), while in the final stage ($t \gg \tau$), Eqs. (6) and (8) are applicable.

Applying a Laplace transform to Eq. (13) and using the convolution theorem, for the image of the temperature T_1 (to avoid confusion the image is denoted by an asterisk superscript) we obtain the problem

$$\frac{d^2 T_1^*}{dx^2} = \frac{1}{\sigma^2} \left(\rho + \sqrt{\frac{\pi \rho}{\theta}} \right) T_1^*, \quad T_1^*|_{x=0} = \frac{T^0}{\rho}, \quad T_1^*|_{x \rightarrow \infty} \rightarrow 0,$$

the solution of which is:

$$T_1^* = \frac{T^0}{\rho} \exp \left[-\frac{x}{\sigma} \left(\rho + \sqrt{\frac{\pi \rho}{\theta}} \right)^{1/2} \right],$$

while it follows from Eq. (12) that

$$T_2^* = \frac{3}{a} \sqrt{\frac{\kappa}{p}} T_1^*.$$

Hence we obtain the image of the thermal flux from the wall to the layer

$$Q^* = -\lambda_* \frac{dT_1^*}{dx} \Big|_{x=0} = \frac{\lambda_* T^0}{\sigma p} \left(p + \sqrt{\frac{\pi p}{\theta}} \right)^{1/2}.$$

Definition of the original quantities then reduces to computation of complex integrals. We will limit ourselves here to obtaining the simplest possible asymptotes. If $t \ll \theta$, then it is admissible to take $\theta p \gg 1$ and use the corresponding series expansions of the images presented above. As a result, for small times we obtain the flux

$$Q \approx T^0 \frac{\lambda_*}{\sigma} \left(\frac{1}{\sqrt{\pi t}} + \frac{1}{2} \sqrt{\frac{\pi}{\theta}} \right), \quad t \ll \theta, \quad (14)$$

and main terms of the temperature expressions

$$T_1 \approx T^0 \operatorname{erfc} \left(\frac{x}{2\sigma \sqrt{t}} \right),$$

$$T_2 \approx \frac{3 \sqrt{\kappa}}{a} T^0 \left[\frac{2}{\sqrt{\pi t}} \exp \left(-\frac{x^2}{4\sigma^2 t} \right) - \frac{x}{\sigma} \operatorname{erfc} \left(\frac{x}{2\sigma \sqrt{t}} \right) \right]. \quad (15)$$

If the layer is gas-filled, we have times satisfying the inequality $\theta \ll t \leq \tau$. In this case we must again use Eqs. (12) and (13), taking $\theta p \ll 1$. In particular, for the thermal flux to the layer we have the expression

$$Q \approx T^0 \frac{\lambda_*}{\sigma} \left(\frac{\pi}{\theta t} \right)^{1/4} \left[\frac{1}{\Gamma(3/4)} + \frac{1}{2\Gamma(1/4)} \sqrt{\frac{\theta}{\pi t}} \right], \quad \theta \ll t \lesssim \tau, \quad (16)$$

where $\Gamma(z)$ is an Euler gamma-function.

For $t \gg \tau$, i.e., in the final stage of the heating process, in place of Eqs. (12) and (13) we must use Eqs. (6) and (8). For simplicity, completely neglecting quantities of the order of τ/t , we obtain expressions corresponding to a one-temperature model of the layer (and a parabolic thermal conductivity equation):

$$Q \approx T^0 \frac{\lambda_*}{\gamma \sqrt{\pi t}}, \quad T_1 \approx T_2 \approx T^0 \operatorname{erfc} \left(\frac{x}{2\gamma \sqrt{t}} \right), \quad t \gg \tau. \quad (17)$$

Thus, heating of a gas-filled granular layer occurs in three clearly distinguishable stages. In the first [Eqs. (14) and (15)] the layer heats up almost as through it were a homogeneous medium with effective heat capacity $\varepsilon d_1 c_1$ per unit volume, of the same order of magnitude as the specific heat of the gas. In the second stage [Eq. (16)] the heat drain to the particles becomes significant; the decrease with time of the heat flux to the layer slows, $Q \sim t^{-1/4}$. Finally, in the third stage [Eq. (17)] thermal equilibrium between the layer phases is practically achieved, the layer again heating up like a homogeneous medium. However, the heat capacity of this medium proves to be equal to the mean heat capacity of the layer itself. If the layer is filled by a droplet liquid then the second stage of the heating process is not realized and there is a continuous transition from the first stage to the third. Roughly speaking, in this case the filled layer heats up like a homogeneous medium, the thermal capacity per unit volume of which increases monotonically from $\varepsilon d_1 c_1$ to dc with increase in t from zero to infinity. Physically, the disappearance of the second stage is related simply to the fact that in this case the particles heat through quite rapidly.

Reliable acquisition of experimental data for times $t \ll \theta$ after commencement of heating of a gas-filled layer is complicated by limited experimental capabilities. From the physical viewpoint there is no doubt whatsoever that at initial times only the gas adjacent to the wall must be heated, as follows from Eq. (14). Over a quite wide time interval, corresponding to the second stage of heating, very much experimental data has been acquired, which are evaluated in [22, 23]. All these data indicate a significant reduction in the rate of de-

crease of flux in this interval. Thus, for example, in [23] Fig. 3 shows data according to which for an increase in Fourier number $Fo \sim \kappa t/a^2$ by three orders of magnitude from 10^{-3} to 1 the Nusselt number, which is proportional to the thermal flux from the wall, decreases by approximately five times. According to the function $Q \sim t^{-1/4}$ reflected by Eq. (16), it should decrease by 5.6 times. On the whole this dependence has been found to be in satisfactory agreement with other experimental facts. In the final stage of the heating process the experimental data are described well by Eq. (17). The disappearance of the second heating stage when the granular layer is filled by droplet liquids has also been confirmed experimentally.

The results obtained for heating of a layer by a solid wall must of course be considered qualitative. They clarify the physical mechanisms of the heat transfer process, but do not conflict with other approaches to the problem. In particular, their derivation has completely neglected the thin interlayer with increased porosity near the wall, for consideration of which it would apparently be necessary to use boundary conditions not of the first, but rather the third sort [29, 30]. Moreover, in the initial moments of heating the inequality $a \ll L$ is obviously disrupted, satisfaction of which is necessary for applicability to the problem under consideration of the continuum methods applied herein. Temperature inhomogeneity along the particle surface mainly affects the values of the effective heat liberation coefficient λ_* of the dispersed system, which quantity experiences frequency dispersion [6, 20]. Therefore, to obtain the corresponding correction in the first approximation we may introduce some mean (over time) quantity λ_*' differing from λ_* , which is to be treated as some empirical parameter. This quantity characterizes effective conductivity of the granular layer in its initial heating stage.

In conclusion, we will note that equations analogous to those above have been obtained for description of problems of great practical import in processes of filling of a fissured-porous medium by a fluid, establishment of a steady-state filtration flow, etc. Due to lack of space these equations cannot be presented here. However, we will note that the data of such processes are also complicated by relaxation phenomena, while for marked non-steady-state conditions fissured-porous media must also be considered as materials with memory.

NOTATION

a , particle size; c_1, c_2, d_1, d_2 , specific heat capacities and densities of materials forming continuous and dispersed phases; L , linear scale of mean fields; ℓ , linear scale of interparticle space; m , coefficient appearing in Eqs. (2) and (7); n , numerical particle concentration; p , Laplace transform variable; Q , thermal flux from wall to granular layer; q , thermal flux to particle; r , radial coordinate; T , temperature within particle; T_1, T_2 , mean temperatures of continuous and dispersed phases; T^0 , wall temperature; t , time; u , mean velocity of continuous phase in spaces between particles; w , effective velocity introduced into Eq. (8); x , coordinate normal to wall; β , interphase heat-exchange coefficient; γ^2, σ^2 , effective thermal conductivity coefficients defined in Eq. (8) and (13); $\delta - \omega' a$; ε , porosity (volume fraction of continuous phase in heterogeneous medium); κ, κ' , thermal diffusivity coefficients of particle and continuous phase materials; λ, λ_* , thermal conductivity coefficients of particles and medium as a whole; τ, θ , time scales introduced in Eqs. (8) and (13); ω , frequency; $\omega' - \sqrt{\omega/2\kappa}$; asterisk superscript denotes Laplace transform.

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